from a very short data sample, the average value should be computed, giving μ_k. Then the standard deviation σ_k should be computed. The worst case distortion parameter can then be calculated for this inlet under these flow conditions from the following relations: subcritical: $K_{\max} = 3\sigma_k + \mu_k$; critical: $K_{\max} = 3.5\sigma_k + \mu_k$; supercritical: $K_{\max} = 5.5\sigma_k + \mu_k$; where K represents this distortion parameter.

Summary

When dealing with random data, which cannot be described with explicit mathematical relations, the methods of statistical analysis can be applied to gain valuable knowledge concerning the characteristics of the data. These methods, with a long history of use in other areas, should be applicable to inlet diagnostics to a much greater degree than in the past.

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Correlation of Turbofan Engine Thrust Performance with Compound Nozzle Flow Theory

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Data from extensive altitude chamber testing of a single nozzle turbofan engine demonstrate the dependence of engine gross thrust on nozzle pressure ratio and simulated flight Mach number (or ram pressure ratio). Variations of gross thrust up to 5% at a constant nozzle pressure ratio are attributed to changes in the engine bypass ratio with power setting. A one-dimensional, compound nozzle flow theory (neglecting mixing) is employed to predict the influence of bypass ratio on the gross thrust of the dual stream nozzle flow. The results emphasize the importance of proper simulation of engine bypass ratio and nozzle pressure ratio in establishing the gross thrust coefficient for inflight thrust evaluation.

Nomenclature

A	= flow area
A_{nozzle}	= total nozzle flow area at given x
$A_P{}^*$	= characteristic area of primary stream, Eq. (16)
A_T	= nozzle exit flow area
B	= fraction of airflow bled at compressor discharge
C_G	= gross thrust coefficient for single stream nozzle flow, Eq. (1)
$C_{G(2)}$	= gross thrust coefficient for dual stream nozzle flow, Eq. (22)
C	= constant
F_G	= gross thrust
$F_{G\text{meas}}$	= measured engine gross thrust
$F_{G\text{theo}}$	= theoretical gross thrust for single stream nozzle flow, Eqs. (2) and (3)
$F_{G\text{theo}(2)}$	= theoretical gross thrust for dual stream nozzle flow, Eq. (12)
f_1,f_2	= functional relationship
h_P	= altitude
H_{TA}	= total enthalpy of air
H_{TG}	= total enthalpy of combustion products
HV	= fuel heating value
M	= Mach number
N_H	= high-pressure compressor rotor speed
P_{TN}	= measured nozzle inlet total pressure (see Fig. 1)
P_T	= total pressure
P	= static pressure
R	= gas constant
T_T	= total temperature
W	= mass flow rate
W_A	= air mass flow rate
W_{ABLD}	= bleed air mass flow rate at compressor discharge
W_{AT}	= total engine air mass flow rate
W_F	= fuel mass flow rate
W_G	= combustion products mass flow rate
X	= dual stream nozzle gross thrust parameter, Eq. (13)
$\%\Delta X/\%\Delta\gamma_P$	= percent change x/percent change γ_P
β	= engine bypass ratio
γ	= ratio of specific heats
Φ	= thrust parameter defined in Eq. (7)
ψ	= nondimensional gross thrust for single stream nozzle flow, Eq. (11)
η_B	= combustor efficiency
θ_T	= corrected total temperature

Subscripts

$(\ )_0$	= simulated freestream flight conditions
$(\ )_{2,2F,3,4,5}$	= engine station designation (see Fig. 1)
$(\ )_e$	= exit plane of converging nozzle
$(\ )_P$	= properties of the primary (turbine discharge) stream
$(\ )_S$	= properties of the secondary (fan discharge) stream
$(\ )_X$	= axial location in the nozzle

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I. Introduction

THE gas generator method has frequently been employed to determine the gross thrust produced by a turbojet or turbofan engine under flight test conditions.[1,2] Parameters measured in-flight are supplemented or modified by known engine characteristics and used to calculate in-flight gross thrust, ultimately to determine net thrust and TSFC. A theoretical gross thrust corresponding to a particular flight test condition is calculated from measured values of the nozzle inlet total pressure, ambient static pressure, and the known nozzle exit area. The actual in-flight gross thrust produced is then determined from the theoretical gross thrust and an empirical gross thrust coefficient. The gross thrust coefficient characteristic of a particular engine-nozzle combination is obtained from extensive wind tunnel and cell tests over the entire range of flight conditions anticipated.

In applying this technique to a turbofan engine which exhausts both the primary and bypass flows through a common nozzle, there is some difficulty in determining the theoretical gross thrust for such a dual stream nozzle flow. Previous studies of similar nozzle flows[3–5] have demonstrated the significant dependence of gross thrust upon the bypass ratio and the degree of mixing between the primary and bypass flows. Consequently, it can be expected that the actual gross thrust of a single nozzle turbofan engine is not only a function of the measured nozzle pressure ratio, but also may depend upon the engine bypass ratio which reflects the distribution of thrust in the dual stream nozzle flow.

Extensive altitude chamber tests of a turbofan engine were conducted to determine the engine gross thrust as a function of power setting, simulated flight Mach number, and altitude, over the range $M_0 = 0$–0.9, $h_P = 0$–$45{,}000$ ft. The results of the turbofan engine thrust testing and the correlation of the data to account for the influence of engine bypass ratio are presented herein.

II. Conventional Correlation of Gross Thrust

A fully instrumented twin-spool turbofan engine in the 10,000 to 15,000 lb_f thrust class was equipped with a conical, converging nozzle and altitude chamber tests were conducted over a range of power setting at the simulated flight conditions shown in Table 1.

A schematic diagram of the engine-nozzle configuration and the engine station designation is shown in Fig. 1. The altitude chamber tests involved the measurement of engine gross thrust, fuel flow, total air flow, and all engine parameters necessary for the evaluation of over-all engine performance. The measured nozzle parameters of interest in the correlation of the gross thrust are P_0, the simulated ambient static pressure, and P_{TN}, the engine discharge or nozzle inlet total pressure. P_{TN} was measured with a production engine probe arrangement that sampled both the turbine and fan discharge flows. Additional parameters measured that are of interest in the evaluation of the engine bypass ratio (see Sec. IV) are the total temperatures T_{T3} and T_{T5} at the discharge of the high-pressure compressor and the low-pressure turbine, respectively.

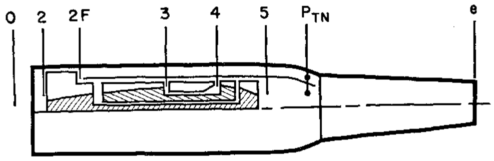

Fig. 1 Schematic of engine-nozzle configuration.

The measured gross thrust, for the range of power setting, altitude, and Mach number tested, was correlated in terms of a gross thrust coefficient defined as

$$C_G = F_{G\text{meas}}/G_{\text{theo}} \tag{1}$$

$F_{G\text{theo}}$, the theoretical gross thrust for a one-dimensional, isentropic, converging nozzle flow, is given as[6]

subcritical flow:

$$F_{G\text{theo}} = A_T P_0 \left\{ \frac{2\gamma}{\gamma - 1} \left[\left(\frac{P_{TN}}{P_0} \right)^{(\gamma-1)/\gamma} - 1 \right] \right\} \tag{2}$$

choked flow:

$$F_{G\text{theo}} = A_T P_0 \left\{ (\gamma + 1) \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma-1)} \frac{P_{TN}}{P_0} - 1 \right\} \tag{3}$$

A nominal, mean value of $\gamma = 1.384$ was used in the calculation of $F_{G\text{theo}}$.

Figure 2 illustrates the correlation of the engine gross thrust data with the gross thrust coefficient presented as a function of the measured nozzle pressure ratio, P_{TN}/P_0. The curves represent the data fairings for tests at constant values of altitude and Mach number (see Table 2 for symbol legend). As can be seen from Fig. 2, the gross thrust coefficient is found to be a function of the nozzle pressure ratio and the simulated flight Mach number or engine face ram pressure ratio P_{T2}/P_0. For a constant nozzle pressure ratio, variations of the gross thrust by as much as 5% were observed for the range of flight conditions simulated. Similar results have been obtained from tests of several different turbofan engines[7,8] and the trends exhibited in Fig. 2 have recently been discussed by Boytos.[2]

The dependence of the turbofan engine gross thrust on the ram pressure ratio can be explained by considering the test conditions employed for obtaining a constant value of nozzle pressure ratio for different simulated flight conditions. The nozzle pressure ratio can be written as

$$P_{TN}/P_0 = P_{TN}/P_{T2} \times P_{T2}/P_0 \tag{4}$$

where P_{TN}/P_{T2} is the engine pressure ratio. To produce a constant nozzle pressure ratio (e.g., a value of 2.5 in Fig. 2) at successively higher values of P_{T2}/P_0, an engine power setting at successively lower values of P_{TN}/P_{T2} is required. For most turbofan engines, such a decrease in the engine power setting generally results in an increase in the engine bypass ratio.

Table 1 Altitude chamber simulated flight conditions[a]

Mach no. M_0	Ram pressure ratio P_{T2}/P_0	Altitude h_P, ft (std. day)
0.0	1.0	0
0.5	1.18	5000, 20,000
0.6	1.27	5000
0.7	1.39	30,000, 40,000, 45,000
0.85	1.60	30,000
0.9	1.70	36,089

[a] The authors gratefully acknowledge the use of preliminary test data provided by the Naval Air Propulsion Test Center, Trenton, N. J. The authors, however, accept full responsibility for the interpretation presented herein.

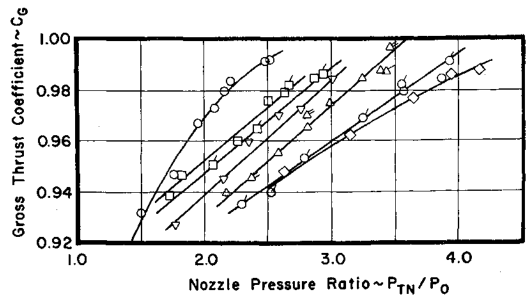

Fig. 2 Variation of gross thrust coefficient with nozzle pressure ratio.

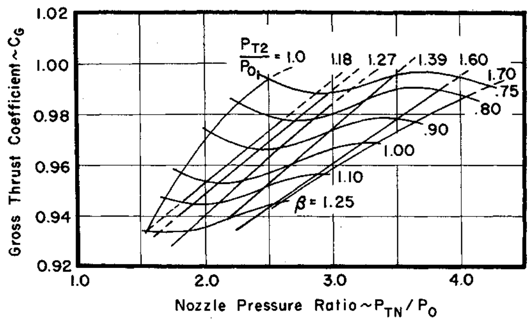

Fig. 3 Variation of gross thrust coefficient with engine bypass ratio.

Thus, the comparison of engine gross thrust data for a given nozzle pressure ratio can involve significant differences in the bypass ratio.

The extent of the variation of the engine bypass ratio β for the subject engine tests is illustrated in Fig. 3. Lines of constant β have been superimposed on the data fairings previously presented in Fig. 2. The values of engine bypass ratio illustrated were calculated from measured engine parameters as described in Sec. IV. For a constant nozzle pressure ratio (e.g., 2.5), an increase in β by as much as 70% is indicated in comparing the data for $M_0 = 0$ and $M_0 = 0.9$. Such variations in the bypass ratio may explain the difference in the turbofan engine gross thrust measured for a given nozzle pressure ratio.

The correlation of the engine test data illustrated in Fig. 2 is completely adequate for calculating in-flight gross thrust for airframe/engine performance evaluation since both P_{TN}/P_0 and P_{T2}/P_0 can be readily determined from conventional flight test data. However, the data presented in Fig. 2 emphasize the importance of proper simulation of the nozzle pressure ratio and the bypass ratio (and primary-bypass mixing characteristics) in the experimental determination of the gross thrust coefficient.[2] This proper simulation is readily accomplished in full scale engine-nozzle tests but requires caution in the use of scale model nozzle test data.

In view of the apparent importance of bypass ratio variations, the analysis and correlation presented in the following sections was developed in the interest of determining the possible quantitative influence of engine bypass ratio on the gross thrust of a single nozzle turbofan engine.

III. Compound Nozzle Flow Theory

The dependence of the gross thrust coefficient C_G upon the ram pressure ratio P_{T2}/P_0 as shown in Fig. 2, can be attributed to the expression employed in evaluating the theoretical gross thrust. As shown in Eqs. (2) and (3), $F_{G\text{theo}}$ is only a function of the nozzle pressure ratio P_{TN}/P_0. From the previous discussion of the apparent influence of engine bypass ratio variations upon the gross thrust of a single nozzle turbofan engine, a more representative expression for evaluating the theoretical gross thrust should be of the form

$$F_{G\text{theo}(2)} = f(P_{TN}/P_0, \beta) \tag{5}$$

There are two relatively simple approaches that can be taken in establishing an expression for calculating $F_{G\text{theo}(2)}$: 1) assume complete mixing of the primary and bypass flows and determine the variation of gross thrust (or thrust augmentation[3,4] as a function of the variation of bypass ratio,[2] or 2) assume negligible mixing of the primary and bypass flows and determine the variation of gross thrust of the dual stream nozzle flow as a function of the variation of bypass ratio.

Although it is frequently considered desirable to completely mix the primary and bypass flows before exhausting through the nozzle,[3-5] most turbofan engines exhibit a variation in the degree of mixing over the range of engine operating conditions. The latter approach indicated previously was adopted herein in investigating the possible influence of bypass ratio variations upon the turbofan engine gross thrust.

Reference 9 presents a compound nozzle flow theory appropriate for the analysis of the flow of two compressible streams in a single converging nozzle. The flow at the nozzle inlet plane, as illustrated in Fig. 4, is characterized in terms of the mass flow, total temperature, and total pressure for 1) the turbine discharge or primary flow W_P, T_{TP}, P_{TP} and 2) the fan discharge or secondary flow W_S, T_{TS}, P_{TS}. The compound flow model is based upon the assumption of one-dimensional, isentropic flow in each stream with no mixing. The interaction between the primary and secondary streams is governed by the assumption of static pressure equilibrium between the streams at any axial position in the nozzle. Recognizing the over-simplification of this model in the neglect of mixing between the primary and secondary streams, the compound nozzle flow theory was employed to compute the theoretical gross thrust of a dual stream, converging nozzle flow for the appropriate engine test conditions.

The gross thrust of the dual stream nozzle flow can be determined by using expressions of the form of Eqs. (2) and (3) to evaluate the gross thrust of both the primary and secondary streams. Thus, for the primary stream the nondimensional gross thrust can be written as

$$\frac{F_{GP}}{A_T P_0} = \left(\left\{2\left(\frac{\gamma_P}{\gamma_P - 1}\right) \times \left[\left(\frac{P_{TP}}{P_e}\right)^{(\gamma_P-1)/\gamma_P} - 1\right] + 1\right\}\frac{P_e}{P_0} - 1\right)\frac{A_P}{A_T} \tag{6}$$

where $(\ )_P$ refers to properties of the primary stream, A_P being the flow area of the primary at the nozzle exit, and P_e the nozzle exit plane static pressure. When the nozzle flow is unchoked, $P_e = P_0$ and Eq. (6) reduces to Eq. (2) written for the primary stream.

It is convenient to define a parameter

$$\Phi = 2[\gamma_P/(\gamma_P - 1)][(P_{TP}/P_e)^{(\gamma_P-1)/\gamma_P} - 1] \tag{7}$$

such that Eq. (6) can be written as

$$F_{GP}/A_T P_0 = [(\Phi_P + 1)(P_e/P_{TP})(P_{TP}/P_0) - 1](A_P/A_T) \tag{8}$$

Expressions similar to Eqs. (6) and (7) can be written for the bypass or secondary stream to yield the following for the nondimension gross thrust of the secondary

$$F_{GS}/A_T P_0 = [(\Phi_S + 1)(P_e/P_{TP})(P_{TP}/P_0) - 1](A_S/A_T) \tag{9}$$

Φ_S is evaluated by using γ_S and P_{TS}/P_e in Eq. (7).

The total nondimensional gross thrust of the dual stream nozzle flow becomes [noting that $(A_P/A_T) + (A_S/A_T) = 1.0$]

$$\frac{F_{G\text{theo}(2)}}{A_T P_0} = \frac{F_{GP}}{A_T P_0} + \frac{F_{GS}}{A_T P_0} = \left[(\Phi_P - \Phi_S)\frac{A_P}{A_T} + \Phi_S + 1\right]\frac{P_e}{P_{TP}}\frac{P_{TP}}{P_0} - 1 \tag{10}$$

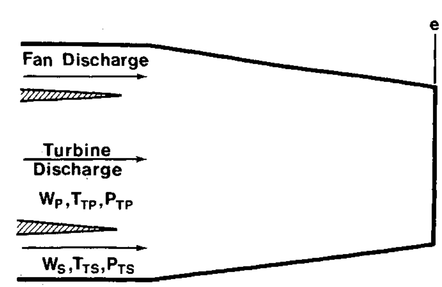

Fig. 4 Compound nozzle flow model.

Upon referring to Eq. (3) it can be seen that the nondimensional gross thrust for a single stream choked nozzle flow can be written as

$$F_{G\text{theo}}/A_T P_0 = \psi(P_{TN}/P_0,\gamma) = C(P_{TN}/P_0) - 1 \quad (11)$$

where the constant C is only a function of γ. Equation (10) shows a similar relationship for the compound nozzle flow. Thus,

$$F_{G\text{theo}(2)}/A_T P_0 = X(P_{TP}/P_0) - 1 \quad (12)$$

where

$$X = [(\Phi_P - \Phi_S)A_P/A_T + \Phi_S + 1]P_e/P_{TP} \quad (13)$$

In the paragraphs that follow, it will be shown that X is also a function of the bypass ratio of the dual stream nozzle flow.

Inspection of Eqs. (7), (12), and (13) indicates that the calculation of $F_{G\text{theo}(2)}$ requires knowledge of the values of P_{TP}/P_e, A_P/A_T, and $P_{TS}/P_e = (P_{TS}/P_{TP})(P_{TP}/P_e)$. The compound nozzle flow theory[4] utilized to obtain this information is described below.

The continuity equation for the primary stream is

$$W_P = A_{PX}P_{TP}(T_{TP})^{-1/2} \times \left(\frac{P_X}{P_{TP}}\right)^{1/\gamma_P} \left\{\frac{2}{R_P}\left(\frac{\gamma_P}{\gamma_P - 1}\right)\left[1 - \left(\frac{P_X}{P_{TP}}\right)^{(\gamma_P-1)/\gamma_P}\right]\right\}^{1/2} \quad (14)$$

where A_{PX} and P_X refer to the flow area of the primary and the static pressure at any x-location in the nozzle. Utilizing a similar expression for W_S of the secondary and noting that

$$A_{\text{nozzle}} = A_{PX} + A_{SX}$$

we obtain the following expression for the compound nozzle flow at the nozzle exit plane (i.e., $A_{\text{nozzle}} = A_T$, $P_X = P_e$)

$$\frac{W_S}{W_P}\left(\frac{T_{TS}}{T_{TP}}\right)^{1/2} = \left\{\frac{A_T}{A_P^*}\left[\left(\frac{2}{\gamma_P - 1}\right) \times \left(\frac{\gamma_P + 1}{2}\right)^{(\gamma_P+1)/(\gamma_P-1)}\right]^{1/2} - \left(\frac{P_{TP}}{P_e}\right)^{1/\gamma_P} \times \left[1 - \left(\frac{P_e}{P_{TP}}\right)^{(\gamma_P-1)/\gamma_P}\right]^{-1/2}\right\} \cdot \left\{\frac{R_P\gamma_S}{R_S\gamma_P}\left(\frac{\gamma_P - 1}{\gamma_S - 1}\right) \times \left[1 - \left(\frac{P_e}{P_{TP}}\frac{P_{TP}}{P_{TS}}\right)^{(\gamma_S-1)\gamma_S}\right]\right\}^{1/2} \left(\frac{P_{TS}}{P_{TP}}\right)\left(\frac{P_e}{P_{TP}}\frac{P_{TP}}{P_{TS}}\right)^{1/\gamma_S} \quad (15)$$

$(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$ is referred to as the temperature corrected bypass ratio. The parameter A_P^* defined as

$$A_P^* = \frac{W_P(T_{TP})^{1/2}}{P_{TP}}\left\{\frac{R_P}{\gamma_P}\left(\frac{\gamma_P + 1}{2}\right)^{(\gamma_P+1)/(\gamma_P-1)}\right\}^{1/2} \quad (16)$$

is characteristic of the primary flow conditions.

When the compound flow is not choked, $P_e = P_0$ and Eq. (15) can be employed to solve for the required value of P_{TS}/P_{TP} for specified values of $(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$, P_{TP}/P_0, A_T, W_P, T_{TP}, P_{TP}, γ_P, γ_S, R_P, and R_S. Thus, Eq. (7) can be utilized to solve for Φ_P and Φ_S. A_P can be obtained directly from Eq. (14) and, therefore, Eq. (10) or Eqs. (12) and (13) can be employed to calculate $F_{G\text{theo}(2)}$ for the specified (unchoked) compound flow conditions.

When the compound nozzle flow is choked an additional relationship is required to determine the flow conditions at the nozzle exit plane. The choking criterion for compound nozzle flow derived in Ref. 4 is analogous to the condition that $M = 1.0$ at the minimum area of a choked, single stream, isentropic nozzle flow. For the dual stream nozzle flow, the choking criterion at the nozzle throat is[4]

$$(A_P/\gamma_P)(1/M_P^2 - 1) + (A_S/\gamma_S)(1/M_S^2 - 1) = 0 \quad (17)$$

Equation (17) shows that for compound choked flow, one of the streams is subsonic at the throat while the other is supersonic.

Utilizing equations of the form of Eq. (14) to eliminate A_P and A_S in Eq. (17) and substituting for M_P and M_S in terms of P_{TP}/P_e and P_{TS}/P_e, the compound choking criterion becomes (at the nozzle throat):

$$\frac{W_S}{W_P}\left(\frac{T_{TS}}{T_{TP}}\right)^{1/2} = \frac{\left(\frac{\gamma_S}{\gamma_P}\frac{P_{TS}}{P_{TP}}\right)\left\{\frac{\gamma_P - 1}{2}\left[\left(\frac{P_e}{P_{TP}}\right)^{(1-\gamma_P)/\gamma_P} - 1\right] - 1\right\}\left(\frac{P_e}{P_{TS}}\right)^{1/\gamma_S}\left\{\frac{2}{R_S}\left(\frac{\gamma_S}{\gamma_S - 1}\right)\left[1 - \left(\frac{P_e}{P_{TS}}\right)^{(\gamma_S-1)/\gamma_S}\right]\right\}^{1/2}}{\left\{1 - \frac{\gamma_S - 1}{2}\left[\left(\frac{P_e}{P_{TS}}\right)^{(1-\gamma_S)/\gamma_S} - 1\right]^{-1}\right\}\left(\frac{P_e}{P_{TP}}\right)^{1/\gamma_P}\left\{\frac{2}{R_P}\left(\frac{\gamma_P}{\gamma_P - 1}\right)\left[1 - \left(\frac{P_e}{P_{TP}}\right)^{(\gamma_P-1)/\gamma_P}\right]\right\}^{1/2}} \quad (18)$$

Equations (15) and (18) provide two independent expres sions for the temperature corrected bypass ratio as a function of P_{TS}/P_{TP}, P_{TP}/P_e and other gas stream properties for compound choked flow. These equations can be employed to determine P_{TS}/P_{TP} and P_{TP}/P_e for specified values of $(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$, A_T/A_P^*, γ_S, γ_P, R_S and R_P. For given flow conditions (which includes a specification of P_{TP}/P_0), the flow will be compound choked if

$$(P_e/P_{TP}) \geq (P_0/P_{TP})$$

When P_{TS}/P_{TP} and P_{TP}/P_e have been determined for a given compound choked flow, Φ_P and Φ_S can be calculated from equations of the form of (7) and A_P can be evaluated from Eq. (14). Thus, the gross thrust for the dual stream, choked nozzle flow can be obtained from Eq. (10).

Examining Eqs. (12) and (13) it can be seen that the functional dependence of the dual stream gross thrust parameter is

$$X = f_1(P_{TP}/P_e, P_{TS}/P_{TP}, A_P/A_T, \gamma_P, \gamma_S)$$

In view of the foregoing discussion, it is apparent that X is a function of the bypass ratio and other gas stream properties.

$$X = f_2[(W_S/W_P)(T_{TS}/T_{TP})^{1/2}, A_P^*/A_T, \gamma_P, \gamma_S, R_P, R_S]$$

A computer program was written for the simultaneous solution of Eqs. (15) and (18) for compound choked flow. Typical results for P_{TS}/P_{TP} as a function of $(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$ and A_P^*/A_T are illustrated in Fig. 5 for nominal values of $\gamma_P = 1.36$, $\gamma_S = 1.4$. Figure 6 illustrates the variation of the nozzle throat static pressure P_e also determined in the solution for compound choked flow. Finally, the variation of the dual stream gross thrust parameter X is shown in Fig. 7. Additional calculations for X for $\gamma_P = 1.34$ and 1.38 indicated a typical variation $(\%\Delta X)/(\%\Delta\gamma_P) = +0.2$ for a constant value of $(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$.

Figure 7, in conjunction with Eqs. (12) and (16), provides a relatively simple method for predicting the gross thrust of a dual stream, unmixed nozzle flow that is similar to the single stream approach [Eq. (3)] frequently employed.

The procedures outlined previously were employed to calculate the dual stream nozzle gross thrust $F_{G\text{theo}(2)}$ for specified values of $(W_S/W_P)(T_{TS}/T_{TP})^{1/2}$, P_{TP}/P_0, W_P, T_{TP}, P_{TP}, γ_P, γ_S, R_P and R_S corresponding to the turbofan engine test conditions previously presented in Sec. II. A comparison of $F_{G\text{theo}(2)}$ and $F_{G\text{meas}}$ is presented in the following section.

IV. Correlation of Gross Thrust with Compound Nozzle Flow Theory

A. Bypass Ratio Calculation

The implementation of the compound nozzle flow theory to determine $F_{G\text{theo}(2)}$ for each engine operating condition re-

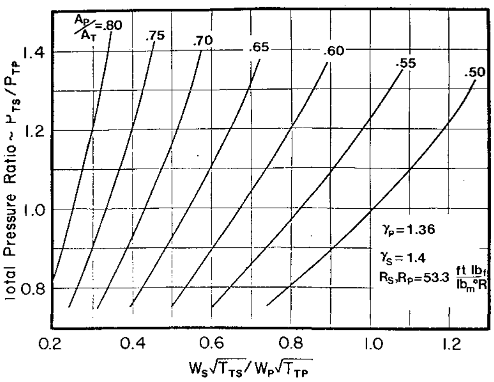

Fig. 5 Total pressure ratio for compound choked flow.

quired, among other things, the value of the engine bypass ratio β. Since the primary and secondary air mass flows were not measured individually, it was necessary to compute a representative value of β for the corresponding test conditions. Previous investigations[7] have shown that this can be accomplished by writing mass and energy balances for the combustor. Thus, using the station designations indicated in Fig. 1 and neglecting the sensible enthalpy of the fuel, we have

$$W_{A3}H_{TA3} + W_F(\eta_B \cdot HV) = W_{G4}H_{TG4} \tag{19}$$

Noting the definitions

$$\beta = \text{bypass ratio} = W_{A2S}/W_{A2P}$$

$$B = \text{bleed ratio} = W_{ABLD}/W_{A2P}$$

$$W_{AT} = W_{A2P} + W_{A2S}$$

the bypass ratio can be written as

$$\beta = (W_{AT}/W_F)[(H_{TG4} - H_{TA3})/(\eta_B \cdot HV - H_{TG4})](1 - B) - 1 \tag{20}$$

The engine parameters measured during the altitude chamber tests provided the necessary values of W_{AT}, W_F, and T_{T3} (for evaluating H_{TA3}) for use in Eq. (20). However, it was necessary to evaluate the turbine inlet condition H_{TG4} from the measured value of turbine discharge total temperature T_{T5} by considering a work balance for the engine. Thus,

$$H_{TG4} = H_{TG5} + [(H_{TA3} - H_{TA2}) + \beta(H_{TA2F} - H_{TA2})]/[1 + (1 + \beta)(W_F/W_{AT}) - B] \tag{21}$$

Equation (21) can be combined with Eq. (20) to obtain a single expression from which the unknown value of β can be determined by iteration.

The following procedure was employed to evaluate β from the altitude chamber test data:

1) Measured values of T_{T5}, T_{T3}, W_F, and W_{AT} were plotted in correction form [e.g., T_{T3}/θ_{T2} vs $N_H/(\theta_{T2})^{1/2}$] to establish a data fairing for each altitude and Mach number tested. Values of each of the parameters read from the *data fairings* at the corresponding value of $N_H/(\theta_{T2})^{1/2}$ were employed in the point by point β calculations to eliminate the extraneous influence of data scatter in the individual parameters.

2) Values for H_{TA3}, H_{TA2}, and H_{TG5} were obtained at T_{T3}, T_{T2}, and T_{T5} from tables of thermodynamic properties for dry air and hydrocarbon-air products of combustion.

3) T_{T2F}, for evaluating H_{TA2F}, was determined from measured values of P_{T2F} and a correlation of T_{T2F}/T_{T2} vs P_{T2F}/P_{T2} developed from published fan performance data.

4) η_β was assumed to be 1.0.

5) Fuel Heating Value = 18,400 B/lb fuel and B = 0.

Figure 8 illustrates the values of β calculated by the aforementioned procedures presented as a function of the engine pressure ratio. The variation of bypass ratio was found to be in good agreement with estimates provided by the engine manufacturer with the calculated values being typically 8 to 3% low as the engine pressure ratio varied from the minimum to the maximum value. The calculated values of β shown in Fig. 8 were employed in the application of the compound nozzle flow theory for the analysis of the dual stream nozzle flow.

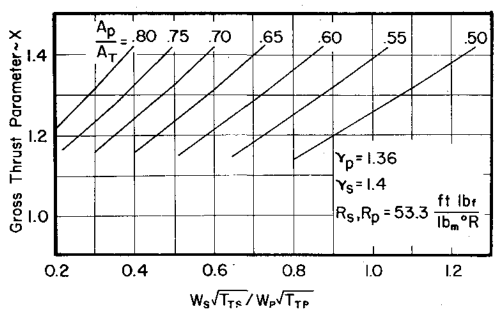

Fig. 7 Gross thrust parameter for compound choked flow.

B. Gross Thrust Correlation

The nozzle inlet conditions for the compound flow gross thrust calculations were evaluated for each engine test condition. The values of primary and secondary stream gas properties employed were: $\gamma_S = 1.4$, $R_S = R_P = 53.35$ (ft lbf/lbm°R), γ_P, evaluated at the mean temperature for the primary flow expansion, ranged from 1.34 to 1.37. The mass flow rate of the secondary was computed as $W_S = W_{AT}[\beta/(1 + \beta)]$, whereas that for the primary was determined as $W_P = W_{AT}/(1 + \beta) + W_F$, neglecting air bleed. The total temperature of the primary was assumed equal to the measured value of turbine discharge total temperature, $T_{TP} = T_{T5}$. For the secondary, it was assumed that $T_{TS} = T_{T2F}$ where T_{T2F} was determined as described in step 3 of the procedures for calculating β. Finally, the primary total pressure P_{TP} was determined from the measured values of P_{TN} and P_{T2F} by assuming that $P_{TS} = P_{T2F}$ and that P_{TN} is a linear average

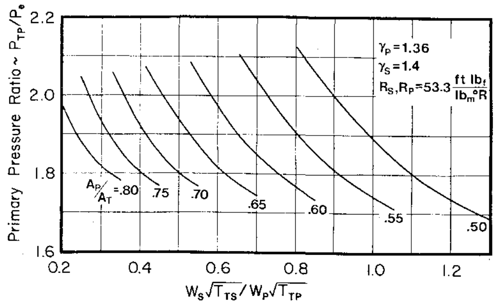

Fig. 6 Primary pressure ratio for compound choked flow.

Table 2 Symbol legend for Figs. 2 and 8–11

Symbol	Mach no.	Ram pressure ratio	Altitude, ft
○	0	1.0	0
□	0.5	1.18	5000
□ (flagged)	0.5	1.18	20,000
▽	0.6	1.27	5000
△	0.7	1.39	20,000
△ (flagged)	0.7	1.39	30,000
△ (double flagged)	0.7	1.39	45,000
○ (flagged)	0.85	1.60	30,000
⬡	0.9	1.70	36,089

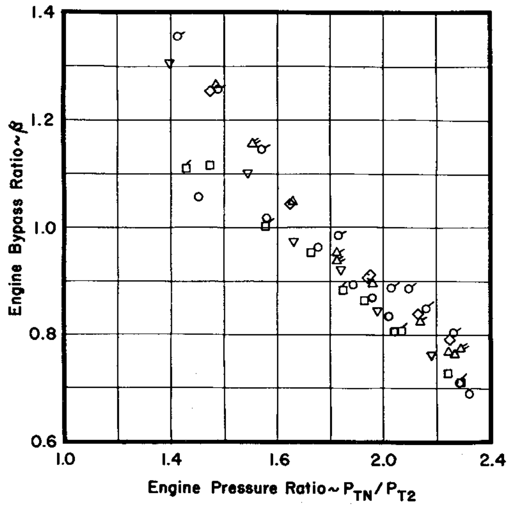

Fig. 8 Engine bypass ratio predicted for engine test conditions.

of P_{TS} and P_{TP}, i.e., $P_{TN} = 0.5(P_{TP} + P_{TS})$. This assumption was found to be in reasonable agreement with published engine performance data.

Using the procedures described previously and in Sec. III, the compound nozzle flow theory was employed to compute the dual stream nozzle gross thrust for each engine test condition illustrated in Sec. II. The results obtained are illustrated in Fig. 9 with the ratio of dual stream to single stream theoretical gross thrust, $F_{G\text{theo}(2)}/F_{G\text{theo}}$, plotted as a function of measured nozzle pressure ratio P_{TN}/P_0 for three values of ram pressure ratio. The plot indicates a variation of the dual stream gross thrust $F_{G\text{theo}(2)}$ with P_{TN}/P_0 and P_{T2}/P_0 very similar to that observed experimentally as was shown in Fig. 2. It can be seen that the utilization of a theoretical gross thrust model that accounts for the influence of engine bypass ratio variations predicts a dependence of gross thrust on bypass ratio that is in good agreement with the trends observed in full scale engine tests. Quantitative comparison of Figs. 9 and 2 indicates the magnitude of the predicted dependence on bypass ratio was somewhat less than the experimental variation. For example, comparing the results for a P_{T2}/P_0 of 1.0 and 1.70 at $P_{TN}/P_0 = 2.5$, one observes approximately a 5% spread in the experimental data in Fig. 2 and a 3% spread in the theoretical predictions in Fig. 9. However, the high-altitude results for $P_{T2}/P_0 = 1.39$ and 1.70 show essentially identical trends in Figs. 2 and 9 with the spread varying from 1 to 2% for a nozzle pressure ratio range of 2.6 to 3.5. Similar results were predicted for the test conditions for the remaining values of P_{T2}/P_0 shown in Fig. 2 but these have been omitted from Fig. 9 for clarity.

The correlation of the measured engine gross thrust with the compound nozzle flow theory is presented in terms of a

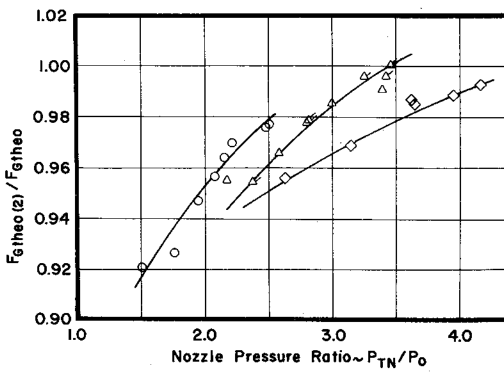

Fig. 9 Variation of theoretical gross thrust ratio with nozzle pressure ratio.

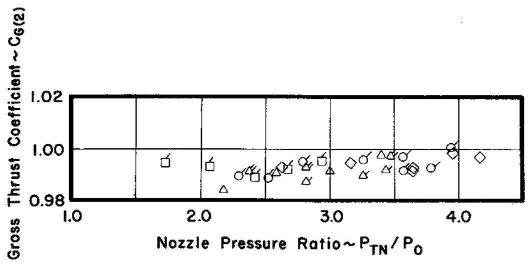

Fig. 10 Variation of compound flow gross thrust coefficient with nozzle pressure ratio (high-altitude data).

dual stream gross thrust coefficient defined as

$$C_{G(2)} = F_{G\text{meas}}/F_{G\text{theo}(2)} \tag{22}$$

The engine test data for simulated altitudes ranging from 20,000 to 45,000 ft are presented in Fig. 10. $C_{G(2)}$ is plotted as a function of P_{TN}/P_0 for a ram pressure ratio range of 1.18 to 1.70. In contrast to Fig. 2, where the high-altitude data for C_G exhibit a variation of 2.5 to 3% for a range of P_{T2}/P_0 of 1.18 to 1.70, the data in Fig. 10 illustrate that $C_{G(2)}$ is essentially independent of P_{T2}/P_0 within a 1% band. It is interesting to note that the value of $C_{G(2)}$ is approximately constant over the range of P_{TN}/P_0.

Figure 11 illustrates a similar plot of $C_{G(2)}$ for the low-altitude data at sea level and 5000 ft. It can be observed that for a given altitude, $C_{G(2)}$ is again approximately constant within a band of 1%. The data at 5000 ft also indicate no significant dependence of $C_{G(2)}$ on P_{T2}/P_0 for the values of 1.18 and 1.27.

There is some increase in the scatter of the $C_{G(2)}$ data for a given value of P_{T2}/P_0 in Figs. 10 and 11 compared to the C_G data in Fig. 2. Furthermore, by comparing Figs. 10 and 11 it can be seen that the level of $C_{G(2)}$ experiences a shift downward with increasing altitude. Although a complete explanation for these trends is not available, it can be expected that definite limitations on the generality of the $C_{G(2)}$ correlation will result from 1) the uncertainty in predicting the value of the engine bypass ratio, 2) the uncertainty of the values of nozzle inlet total pressure and total temperature for the primary and secondary streams, and 3) the neglect of mixing in the compound nozzle flow theory. In spite of these limitations, Figs. 10 and 11 indicate reasonable success in reducing the dependence of the correlation for $F_{G\text{meas}}$ on P_{T2}/P_0. The dependence is essentially eliminated from the high-altitude data and the total spread for all of the data from $P_{T2}/P_0 =$ 1.0 to 1.70 is reduced to approximately 2% compared to the 5% spread shown in Fig. 2.

V. Conclusions

The results of extensive altitude chamber tests of a single nozzle turbofan engine have shown that the engine gross thrust cannot be correlated as a unique function of the nozzle

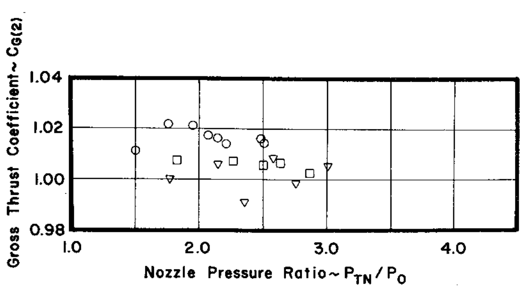

Fig. 11 Variation of compound flow gross thrust coefficient with nozzle pressure ratio (low-altitude data).

pressure ratio. Additional information is required to characterize the distribution of thrust between the primary and bypass flows. The analysis presented herein illustrates that the use of a theoretical expression for gross thrust that accounts for the influence of engine bypass ratio permits the definition of a gross thrust coefficient that is found to be primarily a function of the nozzle pressure ratio. The results of the subject investigation also indicate the potential of the relatively simple compound flow model for use in the development of improved gross thrust coefficient correlations when the availability of full scale engine test data is less extensive.

It should be emphasized that the compound nozzle flow theory employed herein provides only an approximate representation of the flow in a single nozzle turbofan engine exhaust. It can be expected that variations in the degree of mixing between the primary and secondary streams may be of considerable importance. Nevertheless, the application of this rather simplified theoretical model provides added insight into the parameters of importance that are likely to influence the gross thrust of a turbofan engine. It emphasizes the need for accurate data on the engine bypass ratio and the properties of the primary and bypass flows at the nozzle inlet to be obtained in future engine tests or included in the information supplied by the engine manufacturer. Moreover, the importance of proper simulation of the turbofan nozzle flow (pressure ratio, bypass ratio, mixing characteristics, ...) in the experimental determination of the gross thrust coefficient cannot be overemphasized.

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A Comparative Study of Three Axisymmetric Inlets for a Hypersonic Cruise Mission

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The relative merits of three different axisymmetric inlet systems suitable for turboramjet-powered hypersonic cruise vehicles are examined. The differences among the three inlets were centered in their off-design airflow characteristics, and these differences were in turn reflected in the spillage and cowl drags of each inlet. To achieve these different airflow characteristics, two of the configurations utilized a forward translating cowl to achieve a reduction in contraction ratio, and the third utilized a forward translating centerbody. The comparisons were based on the range performance of a blended-body, hydrogen-fueled, Mach number 6.0 cruise vehicle. The forward translating centerbody system was found to be superior. With this system, the vehicle achieved 13.6% greater range than with either of the two forward translating cowl systems. A parametric study was then performed to determine the vehicle's range sensitivity to various inlet performance parameters for the vehicle configured with the forward translating centerbody inlet. The parameters investigated included the inlet total-pressure recovery, the performance of the boundary-layer bleed system, the inlet transonic airflow capability, and the cowl pressure drag. Although no single inlet performance item had a dominant influence on the vehicle's range capability, taken collectively, the effect of improving the various aspects of inlet performance can be substantial.

Nomenclature

A = cross-sectional area
C_{D_a} = additive drag coefficient, D_{add}/q_0A_c
C_{D_c} = cowl drag coefficient, D_{cowl}/q_0A_c
C_{D_s} = spillage drag coefficient, D_{spill}/q_0A_c
D = drag or inlet diameter
L/D = lift to drag ratio
M = Mach number
m = mass flow rate
P_t = total pressure
P_{t_2}/P_{t_0} = inlet total-pressure recovery
q = dynamic pressure
R = range
TOGW = takeoff gross weight
δ = initial exterior cowl angle

Subscripts

bl = bleed
c = cowl or capture
0 = local conditions

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